STAT-101
Chapter 4 Probability
VEREY IMPORTANT

Section 1-4

- **Preview** 'نظرة عامة'

Rare Event Rule for Inferential Statistics: 'الاستنتاجي الإحصائيات'

If, under a given assumption, the probability of a particular observed event is extremely small, we conclude that the assumption is probably not correct. 'احتمال حدوث الحدث المعين الملحوظ صغير جداً ، فإننا نستنتج ان الافتراض قد لايكون صحيحاً'

Statisticians use the rare event rule for inferential statistics.

Section 2-4 Basic Concepts of Probability

- **Key Concept**

This section presents three approaches to finding the probability of an event. ' يعرض هذا القسم ثلاثة أساليب لإيجاد احتمال وقوع الحدث.'

The most important objective of this section is to learn how to interpret probability values. ' الهدف الأهم من هذا القسم هو أن نتعلم كيفية تفسير قيم الاحتمالات'

✓ **Part 1 Basics of Probability**

- **Events and Sample Space**
  - **Event**
    any collection of results or outcomes of a procedure ' أي مجموعة من النتائج أو نتائج إجراءات'
  - **Simple Event**
    an outcome or an event that cannot be further broken down into simpler components. ' نتيجة أو حدث لا يمكن كسر المزيد من ألاسفل إلى مكونات أبسط'
  - **Sample Space important**
    for a procedure consists of all possible simple events; that is, the sample space consists of all outcomes that cannot be broken down any further. ' لإجراء تتألف من جميع الأحداث البسيطة الممكنة؛ وهذا هو، فراغ العينة يتكون من جميع النتائج التي لا يمكن تقسيمها إلى أبعد من ذلك

❖ Notation for Probabilities important

P - denotes a probability. يدل على احتمال

A, B, and C - denote specific events. دلالة على أحداث معينة.

P(A) - denotes the probability of event *A* occurring. يدل على احتمال الحدث A يحدث.

❖ Basic Rules for Computing Probability important

Rule 1: Relative Frequency Approximation of Probability القاعدة الاولى: نسبة تقريب التردد من الاحتمال

Conduct (or observe) a procedure, and count the number of times event *A* actually occurs. Based on these actual results, *P*(*A*) is approximated as follows:

إجراء (أو مراقبة) إجراء، وحساب عدد المرات الحدث A يحدث في الواقع. وبناء على هذه النتائج الفعلية، ويقترب P (A) على النحو التالي:

يحسب عدد المرات الي يمكن يحصل فيها A

$$P(A) = \frac{\text{\# of times } A \text{ occurred}}{\text{\# of times procedure was repeated}}$$

عدد المرات الي يمكن أكرر فيها الإجراء

Rule 2: Classical Approach to Probability (Requires Equally Likely Outcomes) تكون ال events كلها لها احتمالات متساوية الحدوث

Assume that a given procedure has *n* different simple events and that each of those simple events has an equal chance of occurring. If event *A* can occur in *s* of these *n* ways, then

نفترض أن إجراء معين ومختلف n أحداث بسيطة وأن كل تلك الأحداث بسيطة لديه فرصة متساوية من الحدوث. إذا حدث ويمكن أن يحدث في S عن طرق ال n، ثم..

عدد من ways يمكن أن يحدث

$$P(A) = \frac{s}{n} = \frac{\text{number of ways } A \text{ can occur}}{\text{number of different simple events}}$$

عدد مختلف الأحداث بسيطة

Rule 3: Subjective Probabilities قاعدة نظرية

P(*A*), the probability of event *A*, is estimated by using knowledge of the relevant circumstances. ويقدر باستخدام المعرفة من A، واحتمال الحدث P (A)، الظروف ذات الصلة.

مثال على القاعدة رقم ٣:

مثلاً معي نرد له ست اوجهه.. معناه مجموع عدد الاحتمالات = ٦

عندي احتمال ظهور رقم ١ عندي احتمال ظهور رقم ٢ وهكذا ..

احتمال ظهور رقم ١ ~> نقسم الرقم بمجموع عدد الاحتمالات واحد على ستة

❖ Law of Large Numbers =Rule 1

As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability. ' كم يتم تكرار العملية مرارا وتكرارا، واحتمال التكرار النسبي لحدث يميل إلى الاقتراب من احتمال الفعلي. '

❖ **Probability Limits very important**

Always express a probability as a fraction or decimal number between 0 and 1. 'دائما تعبير عن احتمال ككسر أو رقم عشري بين 0 و 1.'

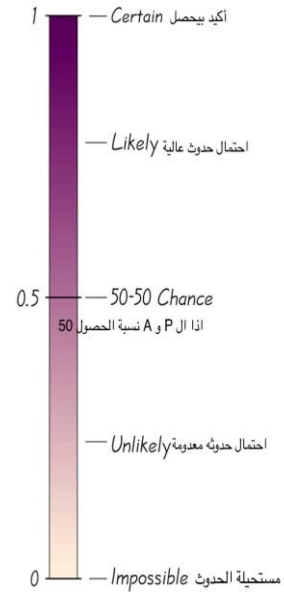
➢ **The probability of an impossible event is 0.** 'احتمال حدوث الحدث المستحيل هو 0.

➢ **The probability of an event that is certain to occur is 1.** 'احتمال حدوث الحدث الذي من المؤكد أن يحدث هو 1

➢ **For any event A, the probability of A is between 0 and 1 inclusive. That is, $0 \leq P(A) \leq 1$.** 'ال "بي" و حرف ال"أ" بتطلع معانا دايم بتتراوح مابين الصفر و الواحد، ف اذا طلعت صفر فهي مستحيلة وإذا طلعت واحد فهي مؤكدة الحدوث'

يوضح النقطة السابقة

ال "P" و ال"A" بتطلع معانا دايم بتتراوح مابين الصفر و الواحد، ف اذا طلعت صفر فهي مستحيلة وإذا طلعت واحد فهي مؤكدة الحدوث



❖ **Complementary Events**

The complement of event A, denoted by $\bar{A}$ BAR, consists of all outcomes in which the event A does not occur. 'استناداً على مثال النرد السابق : مثلاً اريد ان احسب عدم ظهور الإحتمال رقم واحد فستخدم بار مع حرف ال 'أ' ، عدد المرات الي ممكن مايظهر لي الواحد على عدد مجموع الاحتمالات ٦ ، ممكن تظهر ٢ ، ٣ ، ٤ ، ٥ فيساوي عدد عدم احتمالية ظهور الواحد تساوي خمسة'

the rule of complement of event

$$P(\bar{A}) = 1 - P(A)$$

❖ **Rounding Off Probabilities**

When expressing the value of a probability, either give the exact fraction or decimal or round off final decimal results to three significant digits. (Suggestion: When a probability is not a simple fraction such as 2/3 or 5/9, express it as a decimal so that the number can be better understood.). 'عند التعبير عن قيمة الاحتمال، إما إعطاء جزء المحدد أو عشري أو جولة قبالة النتائج العشرية الأخيرة إلى ثلاثة أرقام كبيرة. (اقتراح: عندما الاحتمال ليس جزء بسيط مثل 3/2 أو 9/5، والتعبير عن أنها العشرية بحيث يكون العدد يمكن أن تفهم بشكل أفضل) '

✓ Part 2 Beyond the Basics of Probability: Odds

❖ Odds

The actual odds against event A occurring are the ratio $P(A)/P(\bar{A})$, usually expressed in the form of a:b (or "a to b"), where a and b are integers having no common factors.

احتمالات الفعلية ضد الحدث A تحدث هي نسبة P (A) / P (A)، يعبر عنه عادة في شكل: ب (او "إلى b")، حيث a و b أعداد صحيحة عدم وجود عوامل مشتركة.

The actual odds in favor of event A occurring are the ratio $P(\bar{A})/P(A)$, which is the reciprocal of the actual odds against the event. If the odds against A are a:b, then the odds in favor of A are b:a.

احتمالات الفعلية لصالح الحدث A تحدث هي نسبة P (A) / P (A)، وهو مقلوب على خلاف الفعلية ضد هذا الحدث. إذا كان خلاف ضد وهي: ب، ثم خلاف في صالح مشروع القرار A هي ب: أ.

The payoff odds against event A occurring are the ratio of the net profit (if you win) to the amount bet.

payoff odds against event A = (net profit) : (amount bet)

❖ Recap 'خلاصة'

In this section we have discussed:

- Rare event rule for inferential statistics.
- Probability rules.
- Law of large numbers.
- Complementary events. $P(\bar{A}) = 1 - P(A)$
- Rounding off probabilities.
- Odds. 1. $P(\bar{A})/P(A)$
  2. $P(A)/P(\bar{A})$

مثال من الكتاب للقسم الثاني:

EXAMPLE 13 If you bet \$5 on the number 13 in roulette, your probability of winning is 1/38 and the payoff odds are given by the casino as 35:1.

a. Find the actual odds against the outcome of 13.

b. How much net profit would you make if you win by betting on 13?

c. If the casino was not operating for profit, and the payoff odds were changed to match the actual odds against 13, how much would you win if the outcome were 13?

SOLUTION

a. With $P(13) = 1/38$ and $P(\text{not } 13) = 37/38$, we get

$$\text{actual odds against } 13 = \frac{P(\text{not } 13)}{P(13)} = \frac{37/38}{1/38} = \frac{37}{1} \text{ or } 37{:}1$$

b. Because the payoff odds against 13 are 35:1, we have

$$35{:}1 = (\text{net profit}){:}(\text{amount bet})$$

So there is a \$35 profit for each \$1 bet. For a \$5 bet, the net profit is \$175. The winning bettor would collect \$175 plus the original \$5 bet. That is, the total amount collected would be \$180, for a net profit of \$175.

c. If the casino were not operating for profit, the payoff odds would be equal to the actual odds against the outcome of 13, or 37:1. So there is a net profit of \$37 for each \$1 bet. For a \$5 bet the net profit would be \$185. (The casino makes its profit by paying only \$175 instead of the \$185 that would be paid with a roulette game that is fair instead of favoring the casino.)

Section 3-4 Addition Rule

❖ **Key concept**

This section presents the addition rule as a device for finding probabilities that can be expressed as P(A or B), the probability that either event A occurs or event B occurs (or they both occur) as the single outcome of the procedure. الاثنين يحدثوا مع بعض او احتمالية ظهور واحد منهم

The key word in this section is "or." It is the inclusive or, which means either one or the other or both.

❖ **Compound Event**

any event combining 2 or more simple events 'احتمالية ظهور الاثنين مع بعض'

Notation important P(A or B) = P (in a single trial, event A occurs or event B occurs or they both occur)

❖ **General Rule for a Compound Event**

When finding the probability that event A occurs or event B occurs, find the total number of ways A can occur and the number of ways B can occur, but find that total in such a way that no outcome is counted more than once.

'عند العثور على احتمال أن الحدث A يحدث أو يحدث الحدث B، والعثور على عدد من الطرق يمكن أن يحدث وعدد من الطرق التي يمكن أن تحدث B، ولكن نجد أن المجموع في مثل هذه الطريقة التي لا يتم احتساب نتائج أكثر من مرة'

Formal Addition Rule رح نعرف الفرق بين or , and بعدين

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

عدد احتمالية a,b - عدد ظهور A + عدد ظهور B

where $P(A \text{ and } B)$ denotes the probability that A and B both occur at the same time as an outcome in a trial of a procedure.

Intuitive Addition Rule To find P(A or B), find the sum of the number of ways event A can occur and the number of ways event B can occur, adding in such a way that every outcome is counted only once. P(A or B) is equal to that sum, divided by the total number of outcomes in the sample space.

' للعثور P (A أو B)، والعثور على مجموع عدد من الطرق يمكن أن يحدث الحدث A وعدد من الطرق يمكن أن يحدث الحدث B، مضيفا في مثل هذه الطريقة أن كل نتيجة يتم حساب مرة واحدة فقط. P (A أو B)تساوي هذا المبلغ، مقسوما على إجمالي عدد النتائج في فضاء العينة.'

Rule of Complementary Events

$$P(A) + P(\overline{A}) = 1$$

$$P(\overline{A}) = 1 - P(A)$$

$$P(A) = 1 - P(\overline{A})$$

***P(A)* and *P($\bar{A}$)* are disjoint**

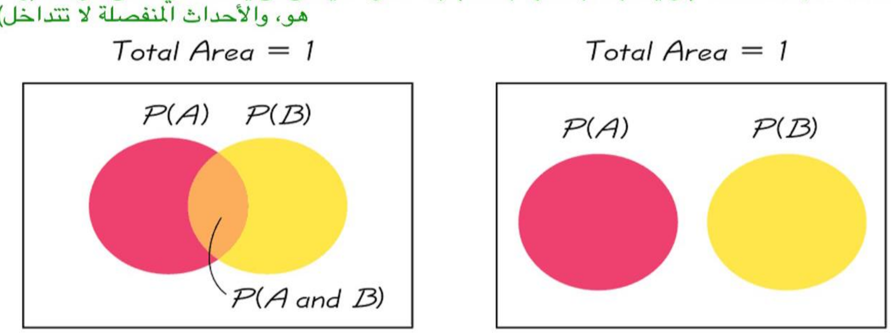
It is impossible for an event and its complement to occur at the same time.

مافيها تقاطع

Disjoint or Mutually Exclusive

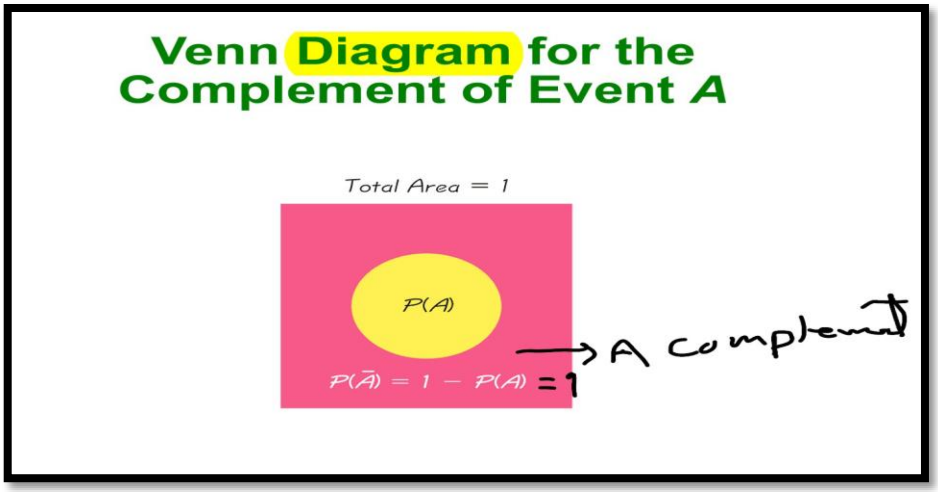
Events *A* and *B* are disjoint (or mutually exclusive) if they cannot occur at the same time. (That is, disjoint events do not overlap.)

أحداث A, B منفصلة (أو يستبعد بعضها بعضا) إذا كانوا لا يمكن أن يحدث في نفس الوقت. (وهذا هو، والأحداث المنفصلة لا تتداخل).



Venn Diagram for Events That Are Not Disjoint

Venn Diagram for Disjoint Events



$$P(A \text{ or } B) = P(A \cap B)$$
$$P(A \text{ or } B) = P(A \cup B)$$

Notation

❖ Recap

In this section we have discussed:

- Compound events.
- Formal addition rule.
- Intuitive addition rule.
- Disjoint events.
- Complementary events.

مثال من الكتاب للقسم الثالث:

EXAMPLE 3 FBI data show that 62.4% of murders are cleared by arrests. We can express the probability of a murder being cleared by an arrest as $P(\text{cleared}) = 0.624$. For a randomly selected murder, find $P(\overline{\text{cleared}})$.

SOLUTION Using the rule of complementary events, we get

$$P(\overline{\text{cleared}}) = 1 - P(\text{cleared}) = 1 - 0.624 = 0.376$$

That is, the probability of a randomly selected murder case *not* being cleared by an arrest is 0.376.

Note:-

$p(\Omega) = 1$

$p(\emptyset) = 0$

Section 4-4 Multiplication Rule: Basics

❖ **Key Concept**

The basic multiplication rule is used for finding P(A and B), the probability that event A occurs in a first trial and event B occurs in a second trial.

'يتم استخدام قاعدة الضرب الأساسية لإيجاد P (A و B)، واحتمال أن الحدث A يحدث في المحاكمة الأولى ويحدث الحدث B في محاكمة ثانية.'

If the outcome of the first event A somehow affects the probability of the second event B, it is important to adjust the probability of B to reflect the occurrence of event A.

' إذا كانت نتيجة الحدث الأول وتؤثر بطريقة أو بأخرى على احتمال الحدث الثاني ب، من المهم لضبط احتمال B لتعكس وقوع الحدث A.'

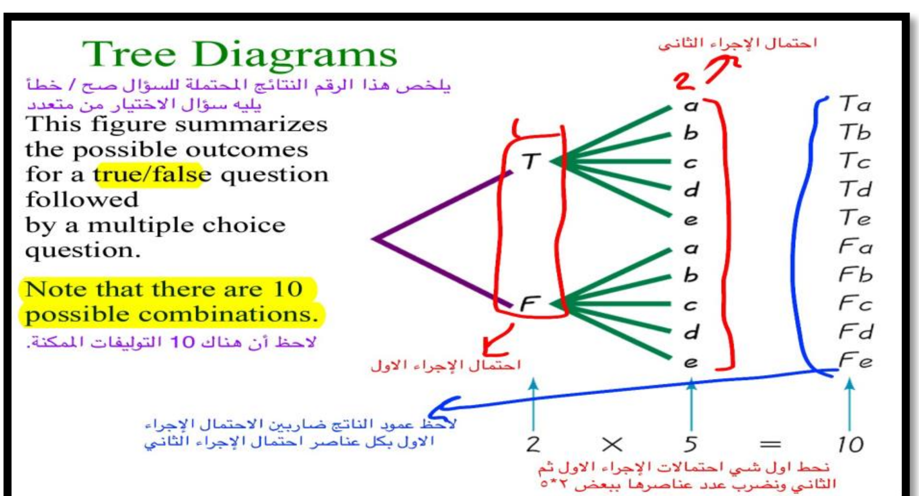
❖ **Notation**

P(A and B) = P(event A occurs in a first trial and event B occurs in a second trial)

❖ **Tree Diagrams**

A tree diagram is a picture of the possible outcomes of a procedure, shown as line segments emanating from one starting point. These diagrams are sometimes helpful in determining the number of possible outcomes in a sample space, if the number of possibilities is not too large.

' هي صورة من النتائج المحتملة لهذا الإجراء، كما هو موضح قطاعات خط المنبثقة عن نقطة انطلاق واحدة. هذه المخططات هي مفيدة أحيانا في تحديد عدد من النتائج المحتملة في فضاء العينة، إذا كان عدد من الاحتمالات ليست كبيرة جدا.'



- **Conditional Probability Key Point important**

  We must adjust the probability of the second event to reflect the outcome of the first event. 'نقطة أساسية يجب علينا ضبط احتمال الحدث الثاني لتعكس نتائج الحدث الأول. '

- **Conditional Probability Important Principle** 'احتمال المشروط مبدأ هام'

  The probability for the second event B should take into account the fact that the first event A has already occurred. ' احتمال لهذا الحدث الثاني "ب "ينبغي أن تأخذ في الاعتبار حقيقة أن أول"أ" حدث وحدث بالفعل. '

- **Notation for Conditional Probability**

  P(B|A) represents the probability of event B occurring after it is assumed that event A has already occurred (read B|A as "B given A.")

- **Dependent and Independent** 'معتمد او غير معتمد'

  Two events A and B are independent if the occurrence of one does not affect the probability of the occurrence of the other. (Several events are similarly independent if the occurrence of any does not affect the probabilities of the occurrence of the others.) If A and B are not independent, they are said to be dependent.

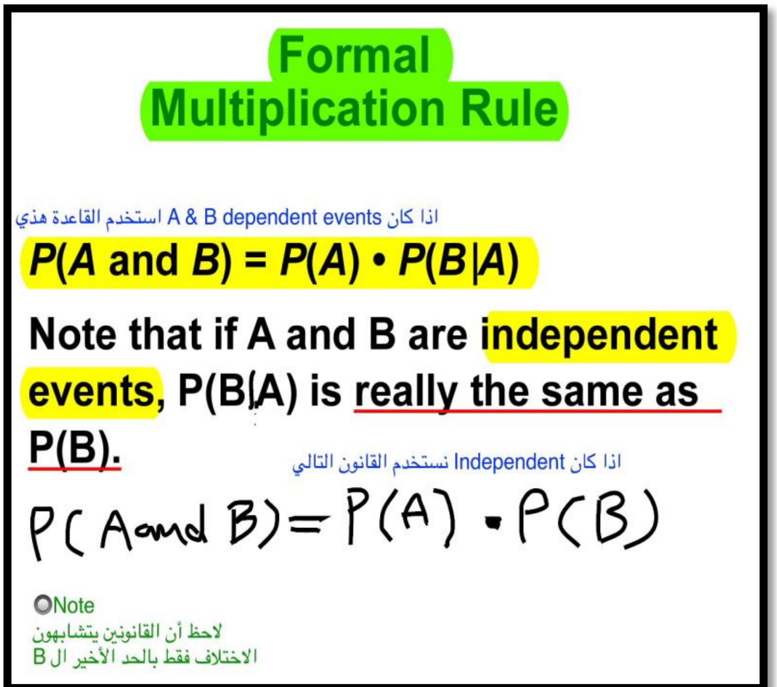
' حدثين A وB إذا كان "انديبنتدنت" وقوع واحدة لا يؤثر على احتمال وقوع الآخر. (العديد من الأحداث بالمثل مستقلة إذا كان وقوع أي لا يؤثر على احتمالات وقوع الاخر.) إذا كان A و B ليست مستقلة، يقال انه "ديبيندينت" . '

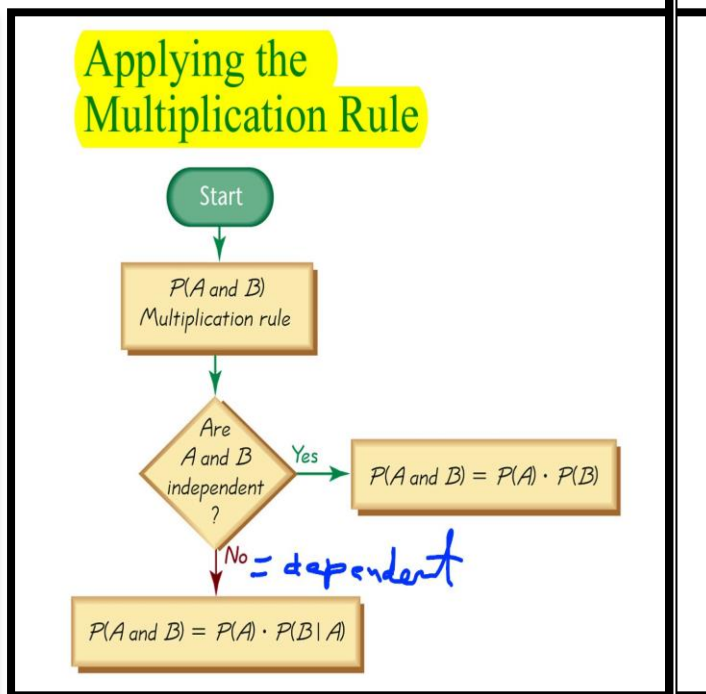
- **Dependent Events**

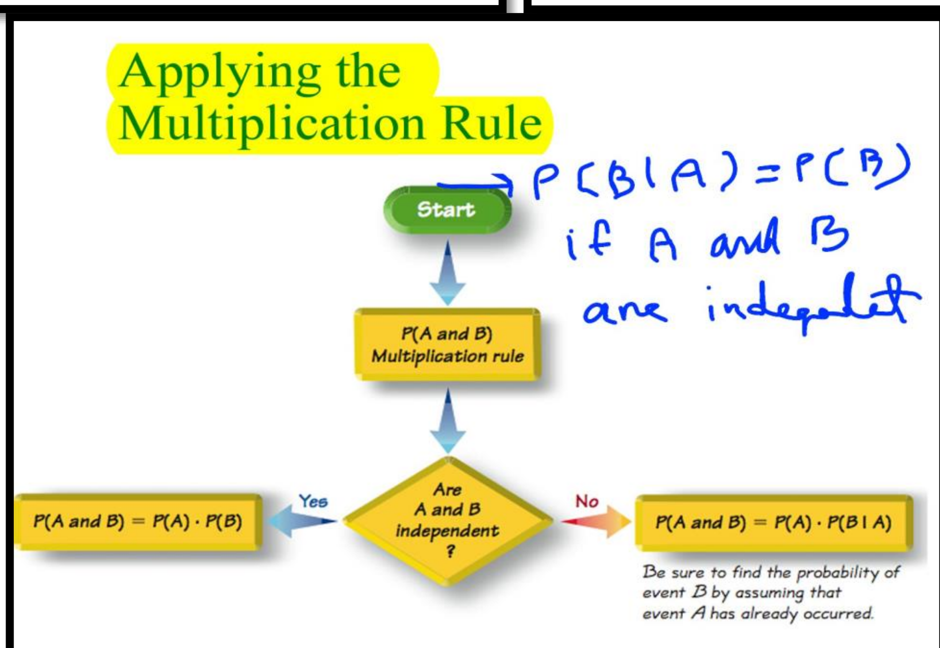
  Two events are dependent if the occurrence of one of them affects the probability of the occurrence of the other, but this does not necessarily mean that one of the events is a cause of the other. ' حدثان تعتمد إذا كان وقوع أحدهما يؤثر على احتمال وقوع الآخر، ولكن هذا لا يعني بالضرورة أن واحدا من الأحداث هو سبب الآخر. '

❖ Intuitive Multiplication Rule

When finding the probability that event A occurs in one trial and event B occurs in the next trial, multiply the probability of event A by the probability of event B, but be sure that the probability of event B takes into account the previous occurrence of event A. ' عند العثور على احتمال أن الحدث 'ا' يحدث في تجربة واحدة ويحدث الحدث 'ب' في التجربة المقبلة، مضاعفة احتمال الحدث 'أ' من احتمال وقوع الحدث 'ب'، ولكن مما لا شك فيه أن احتمال وقوع الحدث 'ب' يأخذ بعين الاعتبار الوجود السابق للحدث 'أ '







- **Caution**

  When applying the multiplication rule, always consider whether the events are independent or dependent, and adjust the calculations accordingly. 'عند تطبيق قاعدة الضرب، انظر دائما ما إذا كانت الأحداث ديبنديت أو اندبيندينت، واضبط الحسابات وفقا لذلك.'

- **Multiplication Rule for Several Events**

  In general, the probability of any sequence of independent events is simply the product of their corresponding probabilities.

  ' بشكل عام، احتمال أي تسلسل الأحداث لل'انديبنيدنت' هو ببساطة نتاج الاحتمالات المقابلة.'

- **Treating Dependent Events as Independent**

  Some calculations are cumbersome, but they can be made manageable by using the common practice of treating events as independent when small samples are drawn from large populations. In such cases, it is rare to select the same item twice.

  ' بعض الحسابات المعقدة يمكن أن يتم التحكم فيها باستخدام الممارسة الشائعة المتمثلة في معالجة الأحداث على أنها 'انديبيندينت' عندما يتم رسمها كعينات صغيرة من عدد كبير من السكان. في مثل هذه الحالات، فمن النادر تحديد العنصر نفسه مرتين. '

- **The 5% Guideline for Cumbersome Calculations**

  If a sample size is no more than 5% of the size of the population, treat the selections as being independent (even if the selections are made without replacement, so they are technically dependent).

  ' إذا كان حجم العينة لا يزيد عن 5٪ من حجم السكان، فانه يتعامل مع التحديدات ال 'اندبينتينت' (حتى إذا تم إجراء التحديدات دون استبدال، بالتالي فهي ديبيندينت من الناحية العملية). '

- **Principle of Redundancy**

  One design feature contributing to reliability is the use of redundancy, whereby critical components are duplicated so that if one fails, the other will work. For example, single-engine aircraft now have two independent electrical systems so that if one electrical system fails, the other can continue to work so that the engine does not fail. ' ميزة تصميم واحد يساهم في موثوقية استخدام التكرار، حيث يتم تكرار العناصر الحاسمة بحيث إذا فشل أحد، والآخر سوف يعمل. على سبيل المثال، طائرة ذات محرك واحد لديها الآن نظامان الكهربائية المستقلة بحيث إذا فشل النظام الكهربائي واحدة، والآخر يمكن أن تستمر في العمل حتى لا تفشل المحرك.
  '

Summary of Fundamentals

- In the addition rule, the word “or” in $P(A \text{ or } B)$ suggests addition. Add $P(A)$ and $P(B)$, being careful to add in such a way that every outcome is counted only once.

في قاعدة الجمع ، فإن كلمة "أو" في P (A أو B) تشير إلى الجمع. إضافة (A) P وP (B)، والحرص على إضافة في مثل هذه الطريقة أن كل نتيجة يتم حساب مرة واحدة فقط.

- In the multiplication rule, the word “and” in $P(A \text{ and } B)$ suggests multiplication. Multiply $P(A)$ and $P(B)$, but be sure that the probability of event B takes into account the previous occurrence of event A.

في قاعدة الضرب، كلمة "و" في P (A و B) تشير إلى الضرب. تتضاعف (A) P و(B) P، ولكن مما لا شك فيه أن احتمال وقوع الحدث B يأخذ بعين الاعتبار حدوث السابق من الحدث A.

Recap

In this section we have discussed:

- Notation for $P(A \text{ and } B)$.
- Tree diagrams.
- Notation for conditional probability.
- Independent events.
- Formal and intuitive multiplication rules.

مثال من الكتاب للقسم الرابع:

EXAMPLE 2 **Quality Control in Manufacturing** Pacemakers are implanted in patients for the purpose of stimulating pulse rate when the heart cannot do it alone. Each year, there are more than 250,000 pacemakers implanted in the United States. Unfortunately, pacemakers sometimes fail, but the failure rate is low, such as 0.0014 per year (based on data from "Pacemaker and ICD Generator Malfunctions," by Maisel, et al., *Journal of the American Medical Association*, Vol. 295, No. 16). We will consider a small sample of five pacemakers, including three that are good (denoted here by G) and two that are defective (denoted here by D). A medical researcher wants to randomly select two of the pacemakers for further experimentation. Find the probability that the first selected pacemaker is good (G) and the second pacemaker is also good (G). Use each of the following assumptions.

a. Assume that the two random selections are made *with replacement,* so that the first selected pacemaker is replaced before the second selection is made.

b. Assume that the two random selections are made *without replacement,* so that the first selected pacemaker is *not* replaced before the second selection is made.

SOLUTION Before proceeding, it would be helpful to visualize the three good pacemakers and the two defective pacemakers in a way that provides us with greater clarity, as shown below.

G G G D D

a. If the two pacemakers are randomly selected *with replacement*, the two selections are independent because the second event is not affected by the first outcome. In each of the two selections there are three good (G) pacemakers and two that are defective (D), so we get

$$P(\text{first pacemaker is G and second pacemaker is G}) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25} \text{ or } 0.36$$

b. If the two pacemakers are randomly selected *without replacement*, the two selections are dependent because the probability of the second event is affected by the first outcome. In the first selection, three of the five pacemakers are good (G). After selecting a good pacemaker on the first selection, we are left with four pacemakers including two that are good. We therefore get

$$P(\text{first pacemaker is G and second pacemaker is G}) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} \text{ or } 0.3$$

Section 4-5 Multiplication Rule: Complements and Conditional Probability

❖ **Key Concept**

Probability of "at least one":

Find the probability that among several trials, we get at least one of some specified event.

' احتمال "واحد على الأقل": العثور على احتمال أن من بين العديد من التجارب، الحصول على واحد على الأقل من بعض الأحداث المحددة. '

Conditional probability:

Find the probability of an event when we have additional information that some other event has already occurred.

' الاحتمال الشرطي: البحث عن احتمال وقوع الحدث عندما يكون لدينا معلومات إضافية بينما بعض الأحداث الأخرى قد حدثت بالفعل. '

❖ **Complements: The Probability of "At Least One"**

- **"At least one" is equivalent to "one or more."** ' واحد على الأقل "ما يعادل" واحد أو أكثر. '
- **The complement of getting at least one item of a particular type is that you get <u>no</u> items of that type.**

Conditional Probability

A conditional probability of an event is a probability obtained with the additional information that some other event has already occurred. $P(B|A)$ denotes the conditional probability of event B occurring, given that event A has already occurred, and it can be found by dividing the probability of events A and B both occurring by the probability of event A:

قانون تكلمنا عنه سابقاً → $P(A \text{ and } B) = P(A) \cdot P(B|A)$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Finding the Probability of "At Least One"

To find the probability of at least one of something, calculate the probability of none, then subtract that result from 1. That is,

للعثور على احتمال واحد على الأقل من "شيء"، وحساب الاحتمالات من "لا شيء"، اطرح تلك النتيجة من 1.

$$P(\text{at least one}) = 1 - P(\text{none}).$$

❖ **Intuitive Approach to Conditional Probability**

The conditional probability of B given A can be found by assuming that event A has occurred, and then calculating the probability that event B will occur.

' الاحتمال الشرطي من A B معين يمكن العثور على افتراض أن الحدث A حدث، ومن ثم حساب احتمال سيحدث هذا الحدث B. '

Confusion of the Inverse

To incorrectly believe that $P(A|B)$ and $P(B|A)$ are the same, or to incorrectly use one value for the other, is often called confusion of the inverse.

الاعتقاد غير صحيح أن P (A | B) وP (B | A) هي نفسها، أو لاستخدام قيمة واحدة غير صحيح للآخر، وغالبا ما تسمى الخلط بين معكوس.

$$P(B|A) \neq P(A|B)$$

Recap

In this section we have discussed:

- Concept of "at least one."
- Conditional probability.
- Intuitive approach to conditional probability.

Omitting sec 6,7,8 .. End of chapter 4

مثال من الكتاب للقسم الخامس

EXAMPLE 2 **Defective Firestone Tires** Assume that the probability of a defective Firestone tire is 0.0003 (based on data from Westgard QC). If the retail outlet CarStuff buys 100 Firestone tires, find the probability that they get at least 1 that is defective. If that probability is high enough, plans must be made to handle defective tires returned by consumers. Should they make those plans?

SOLUTION

Step 1: Use a symbol to represent the event desired. In this case, let A = at least 1 of the 100 tires is defective.

Step 2: Identify the event that is the complement of A.

$$\begin{aligned}\overline{A} &= \textit{not} \text{ getting at least 1 defective tire among 100 tires}\\ &= \text{all 100 tires are good}\\ &= \text{good and good and} \ldots \text{and good (100 times)}\end{aligned}$$

Step 3: Find the probability of the complement.

$$\begin{aligned}P(\overline{A}) &= 0.9997 \cdot 0.9997 \cdot 0.9997 \cdot \cdots \cdot 0.9997 \text{ (100 factors)}\\ &= 0.9997^{100} = 0.9704\end{aligned}$$

Step 4: Find $P(A)$ by evaluating $1 - P(\overline{A})$.

$$P(A) = 1 - P(\overline{A}) = 1 - 0.9704 = 0.0296$$

INTERPRETATION There is a 0.0296 probability of at least 1 defective tire among the 100 tires. Because this probability is so low, it is not necessary to make plans for dealing with defective tires returned by consumers.

Examples:

Example 7.1

Consider the experiment of rolling a fair die once.

a) What is the probability of getting a SIX on one throw?

Solution

Let A be the event of getting a SIX on one throw. The sample space is $S = \{1,2,3,4,5,6\}$ and $n(A) = 1$. Hence,

$$P(A) = \frac{n(A)}{N} = \frac{1}{6} = 0.1667$$

b) What is the probability of getting an even number?

Solution

Define A to be the event of obtaining an even number. The even numbers are 2, 4, and 6, therefore, $n(A) = 3$.

$$P(A) = \frac{n(A)}{N} = \frac{3}{6} = 0.5$$

Example 7.2

A pair of coin is tossed once.

a) What is the probability of getting one HEAD (H) only?

Solution

$S = \{(HH),(HT),(TT),(TH)\}$. The sample space has four elements, $N = 4$.

Define A to be the event of getting one H only. Since there are only two instances when H appears once, $n(A) = 2$.

$$P(A) = \frac{n(A)}{N} = \frac{2}{4} = 0.5$$

b) What is the probability of getting at least one TAIL (T)?

Solution

Define A to be the event of getting at least one T. "At least one T" means one T or two Ts. Getting back at the sample space we see that $n(A)=3$.

$$P(A)=\frac{n(A)}{N}=\frac{3}{4}=0.75$$

Relative Frequency Method

Other books call relative frequency method as the empirical or a posteriori probability. If A is the event being investigated, and N is the number of times the experiment is repeated then

$$P(A)=\frac{n(A)}{N}$$

$n(A)$ is the number of times event A occurred.

Example 7.3

Consider a teacher who asks his students to think of a number from among 1, 2, 3, 4, 5. Students wrote down the number on a piece of paper and the table below shows the results.

Number	Frequency
1	7
2	15
3	10
4	5
5	3

7.3 FUNDAMENTAL RULES OF PROBABILITY

Addition Rule

If A and B are any two events then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example 7.9

Consider the experiment of drawing a card on a deck of 52 playing cards. What is the probability of drawing a red card or a face card?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52}$$

$$= \frac{8}{13}$$

Addition Rule Mutually Exclusive Events

If A and B are any two mutually exclusive events then $P(A \cup B) = P(A) + P(B)$

Example 7.10

What is the probability of drawing a king of heart or a queen of spades.

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{1}{52} + \frac{1}{52} = \frac{2}{52}$$

$$P(A \cup B) = \frac{1}{26}$$

Example 7.11

What is the probability of drawing a card numbered 8 or black card?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{4}{52} + \frac{26}{52} - \frac{2}{52}$$

$$= \frac{28}{52}$$

$$P(A \cup B) = 14/26$$

سلايدات مس أبها فرع الدمام :

Relapse

Researchers randomly assigned 72 chronic users of cocaine into three groups: desipramine (antidepressant), lithium (standard treatment for cocaine) and placebo. Results of the study are summarized below.

بشكل عشوائي 72 مدمن من الكوكايين إلى ثلاث مجموعات: ديسيبرامين (اكتئاب)، والليثيوم (العلاج القياسي للكوكايين) الوهم. وفيما يلي ملخص نتائج الدراسة.

	relapse	no relapse	total
desipramine	10	14	24
lithium	18	6	24
placebo	20	4	24
total	48	24	72

http://www.oswego.edu/~srp/stats/2_way_tbl_1.htm

Marginal probability

What is the probability that a patient relapsed?

ما هو احتمال أن المريض انتكس؟

	relapse	no relapse	total
desipramine	10	14	24
lithium	18	6	24
placebo	20	4	24
total	48	24	72

P(relapsed) = 48 / 72 ~ 0.67

Joint probability

What is the probability that a patient received the antidepressant (desipramine) and relapsed?

ما هو احتمال أن المريض تلقى المضادة للاكتئاب (ديسيبرامين) وانتكس؟

	relapse	no relapse	total
desipramine	10	14	24
lithium	18	6	24
placebo	20	4	24
total	48	24	72

P(relapsed and desipramine) = 10 / 72 ~ 0.14

Conditional probability

The conditional probability of the outcome of interest A given condition B is calculated as يتم حساب الاحتمال الشرطي لنتائج الفائدة ل A نظرا لحالة B

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

	relapse	no relapse	total
desipramine	10	14	24
lithium	18	6	24
placebo	20	4	24
total	48	24	72

$$P(relapse|desipramine) = \frac{P(relapse \text{ and } desipramine)}{P(desipramine)}$$

$$= \frac{10/72}{24/72}$$

$$= \frac{10}{24}$$

$$= 0.42$$

Conditional probability (cont.)

وإذا كنا نعرف أن المريض تلقى مضاد للاكتئاب (ديسيبرامين)، ما هو احتمال أنهم انتكسوا؟

If we know that a patient received the antidepressant (desipramine), what is the probability that they relapsed?

	relapse	no relapse	total
desipramine	10	14	24
lithium	18	6	24
placebo	20	4	24
total	48	24	72

P(relapse | desipramine) = 10 / 24 ~ 0.42

Conditional probability (cont.)

If we know that a patient received the antidepressant (desipramine), what is the probability that they relapsed?

	relapse	no relapse	total
desipramine	10	14	24
lithium	18	6	24
placebo	20	4	24
total	48	24	72

P(relapse | desipramine) = 10 / 24 ~ 0.42

P(relapse | lithium) = 18 / 24 ~ 0.75

P(relapse | placebo) = 20 / 24 ~ 0.83

Conditional probability (cont.)

If we know that a patient relapsed, what is the probability that they received the antidepressant (desipramine)?

	relapse	no relapse	total
desipramine	10	14	24
lithium	18	6	24
placebo	20	4	24
total	48	24	72

P(desipramine | relapse) = 10 / 48 ~ 0.21

P(lithium | relapse) = 18 / 48 ~ 0.38

P(placebo | relapse) = 20 / 48 ~ 0.42

General multiplication rule

- Earlier we saw that if two events are independent, their joint probability is simply the product of their probabilities. If the events are not believed to be independent, the joint probability is calculated slightly differently.

رأينا سابقا أنه إذا حدثين هو independent ، احتمال المشترك هو ببساطة نتاج احتمالات حصولها. إن لم يكن يعتقد أن الأحداث أن يكون independent ، يتم حساب الاحتمال المشترك بشكل مختلف قليلا.

- If A and B represent two outcomes or events, then

  P(A and B) = P(A | B) x P(B) إذا A و B تمثل اثنين من النتائج أو الأحداث، إذاً..

- Note that this formula is simply the conditional probability formula, rearranged. لاحظ أن هذه الصيغة هي مجرد صيغة الاحتمال المشروط، وترتيبها.

- It is useful to think of A as the outcome of interest and B as the condition. ومن المفيد أن نفكر في A كنتيجة مفيدة وB كشرط.

Independence and conditional probabilities

Consider the following (hypothetical) distribution of gender and major of students in an introductory statistics class:

انظر في (افتراضية) التوزيع التالي من الجنسين، وتخصص للطلاب في فئة إحصاءات التمهيدية:

	social science	non-social science	total
female	30	20	50
male	30	20	50
total	60	40	100

احتمال أن الطالب تم اختياره عشوائيا و تخصصة العلوم الاجتماعية =

- The probability that a randomly selected student is a social science major is 60 / 100 = 0.6.
- The probability that a randomly selected student is a social science major given that they are female is

  احتمال أن طالب تم اختياره عشوائيا وتخصصه علوم الاجتماعية بشرط يكون الجنس أنثى=

  30 / 50 = 0.6.
- Since P(SS | M) also equals 0.6, major of students in this class does not depend on their gender: P(SS | F) = P(SS).

Independence and conditional probabilities (cont.)

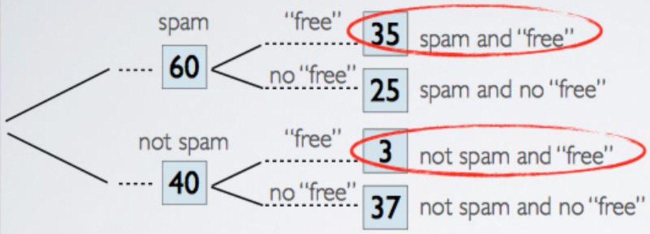
Generically, if P(A | B) = P(A) then the events A and B are said to be independent.

- Conceptually: Giving B doesn't tell us anything about A.
- Mathematically: We know that if events A and B are independent, P(A and B) = P(A) x P(B). Then,

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$$

Probability Trees

You have 100 emails in your inbox: 60 are spam, 40 are not. Of the 60 spam emails, 35 contain the word "free". Of the rest, 3 contain the word "free". If an email contains the word "free", what is the probability that it is spam?



$$P(\text{spam} \mid \text{"free"}) = \frac{35}{35+3} = 0.92$$

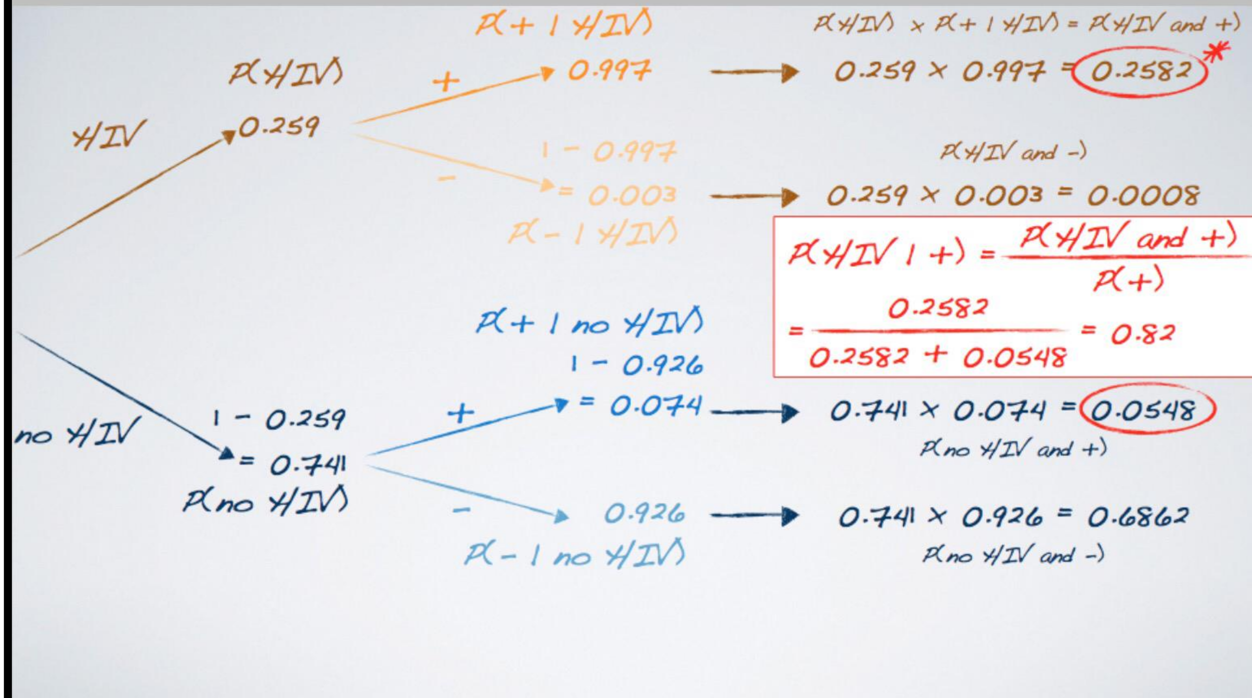
As of 2009, Swaziland had the highest HIV prevalence in the world. 25.9% of this country's population is infected with HIV. The ELISA test is one of the first and most accurate tests for HIV. For those who carry HIV, the ELISA test is 99.7% accurate. For those who do not carry HIV, the test is 92.6% accurate. If an individual from Swaziland has tested positive, what is the probability that he carries HIV?

$P(HIV) = 0.259$

$P(+ \mid HIV) = 0.997$ $\quad P(- \mid \text{no } HIV) = 0.926$

tree diagram!

$P(HIV \mid +) = ?$



إذا كان هناك شخص من سوازيلاند ونتائج اختبار مرضه إيجابي، ما هو احتمال أن يحمل فيروس نقص المناعة البشرية؟

If an individual from Swaziland has tested positive, what is the probability that he carries HIV?

$P(HIV \mid +) = 0.82$

هناك فرصة 82٪ فرد من سوازيلاند نتائجهم جاءت ايجابية فعلا يحمل فيروس نقص المناعة البشرية.

There is an 82% chance that an individual from Swaziland who has tested positive actually carries HIV.

سؤالين إضافيين مع شرح بسيط لهم

- Let the sample spay Ω= {a, b, c, d, e, f, j } A= {a, b, c} and B= {a, b ,e} find the following:-
  a) P(A) = $\frac{3}{7}$
  b) P(A ∩ B) → $\frac{1}{7}$
  c) P(A U B) → $\frac{4}{7}$
  d) P(A/ B) → $\frac{1}{3}$
  e) P ($\overline{A}$) → $\frac{4}{7}$

Answers:

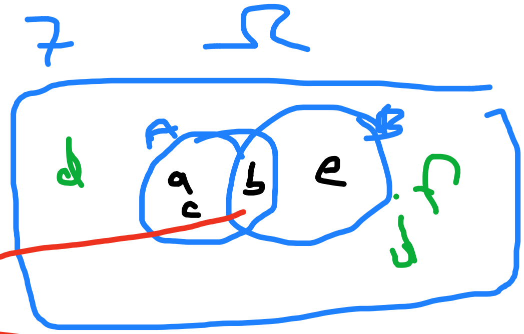
a) ⇒ $\frac{\text{عدد احتمالية حدوث A}}{\text{عدد الاحتمالات}} = \frac{3}{7}$ → A, → Ω

طريقة الأولى

b) → P(A and B) = P(A∩B) → $\frac{1}{7}$

طريقة ثانية

→ Venn Diagram →



1 = b احتمالية

d) → P(A and B) = P(A) · P(B/A) →

$$P(A/B) = \frac{P(A \text{ and } B)}{P(A)}$$

$$\frac{1/7}{3/7} = \frac{1}{7} = \frac{7}{3} = \frac{1}{3}$$

c) → $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$= \frac{3}{7} + \frac{2}{7} - \frac{1}{7} = \frac{4}{7}$$

e) → $1 - P(A)$

$$= 1 - \frac{3}{7} = \frac{4}{7}$$

❖ **Let P(B)= 0.3, P(A)= 0.2, if A and B are independent, then P(A ∩ B)** = P(A and B)

a) **0.5**
b) **0.06** ✓
c) **0.3**
d) **0.2**

$P(A/B) = P(A) = 0.2$

$P(A \cap B) = 0.2 \times 0.3 = 0.06$